Ground states and a-priori bounds for solutions
of nonlinear elliptic equations involving a gradient term

Marta García-Huidobro
Pontificia Universidad Católica de Chile

The purpose of this talk is to give some results concerning a-priori bounds and existence / nonexistence of positive solutions in a domain $\Omega \subseteq \mathbb{R}^N$ of the equations

$$-\Delta u = |u|^{p-1}u|\nabla u|^q \tag{1}$$

where $p + q > 1$ and

$$-\Delta u = |u|^{p-1}u + M|\nabla u|^q, \tag{2}$$

where $p > 1$, $q > 1$ and $M \in \mathbb{R}$.

We address the questions of

- Upper estimates of solutions
- Existence or nonexistence of solutions in all $\mathbb{R}^N$.

For problem (1), using a direct Bernstein method we obtain a first range of values of p and q in which $u(x) \leq c(\mathrm{dist}(x, \partial\Omega)^{\frac{q-2}{p+q-1}}$ This holds in particular if $p + q < 1 + \frac{4}{N-1}$. Using an integral Bernstein method we obtain a wider range of values of p and q in which all the global solutions are constants. Our result contains Gidas and Spruck nonexistence result as a particular case. For problem (2) we completely classify the radial solutions depending on the parameter M in the critical casef $q = \frac{2p}{p+1}$.